BAV Project

**Model estimation**

**Model from first-round paper**

**My suggestion for an MNL Market share model**

1. Following Cooper and Nakanishi (2010, p. 128), the attraction Abt for brand b in period t and its associated market share sbt in an MNL model are given by

(1)

and

(2)

where αb is brand-specific intercept, κbt are *year-* and brand-specificfixed effects (starting in year 2, i.e. ,month 13), *Xbkt* is the matrix of explanatory variables (price, promotion, distribution, and ad-stock) with heterogenous response coefficients, Wbl is the matrix of brands’ attribute levels with homogenous response coefficients, εbt is the error term, and m is the number of all available brands b in time period t.

1. Linearizing equation (1) yields

(3)

1. Introduction of endogeneity

Gaussian copulas are used to model the correlation between the potentially endogenous regressors contained in X (Pricebt, Promotionbt, Distributionbt, and AdStockbt), and the error term . Following Park and Gupta (2012), we compute control variables, and add them to the matrix X. We apply the following transformation (see footnote 3, p. 572 in Park and Gupta 2012): X\*bkt=, where is the inverse distribution function of the standard normal, and H(·) is the empirical cumulative distribution function of Xbk (or of Xk, discuss this with Harald).

1. We follow the derivations in Cooper and Nakanishi (2010), and first take the log of (2), and then plug in (4) to yield

(4)

or, equivalently

(5)

1. Following Cooper and Nakanishi (2010, formulas 5.6 and 5.14), we apply a transformation by the geometric average brand: to do this, we sum (5) over all b, and divide by m (the number of brands in period t).

(6)

where , i.e., the geometric average.

1. Subtracting (6) from (5) yields the model to be estimated, see also formula 5.14 in Cooper and Nakanishi (2010, p. 128).

(7)

where dj is 1 if b=j, and 0 otherwise. α1 is not estimated to ensure identification.

Estimation either via SUR, or Maximum Likelihood (better for model extensions, see below).

**Extensions**

1. In a later stage, we can make and a function of CBBE, e.g.

(8)

(9)

For identification, α0,j=1 and are set to zero.

1. Alternatively, a HLM model can be obtained by adding random components to and in (8) and (9)

(10)

(11)

where and